Journal of Advanced Studies: Sciences,Technologies,
Humanities, Arts.
ISSN: Pending Volumen: 2 Issue: 2
Published online on September, 05, 2025.

Computational Simulation of Moduli Spaces and Radon Transforms Applied to Black Hole–Type Fields

Jesús Francisco Carpio-Mendoza[1]

Abstract

This work presents the development of a computational model in R for the analysis and comparison of moduli spaces inspired by cohomological relations of black hole–type fields. Discrete Radon transforms and Lagrangian operators are implemented to identify invariants stable under broad resolutions, emulating concepts derived from the Penrose transform.

The methodology includes the construction of a discrete grid, the generation of a field with smoothed singularities and annular structures, the application of gradient and Laplacian operators, and the extraction of computational invariants through the 2D Radon transform. The results demonstrate the stability of cohomological invariants after Gaussian smoothing, validating the equivalence of moduli spaces at different resolutions.

The research is grounded in the work of Dr Francisco Bulnes, in particular Penrose Transform on D-Modules, Moduli Spaces and Field Theory (Advances in Pure Mathematics, 2012).

keywords: Moduli spaces, Radon transform, Lagrangian operators, Black hole–type fields, Penrose transform, Cohomology, Computational models, Mathematical physics.
AMS Classification.5808, 5809

1 Introduction

The study of moduli spaces and their cohomological relations has gained relevance in mathematical physics and field theory, especially in contexts related to black holes and complex geometries.

The Radon transform, by enabling the projective integration of functions over hyperplanes, provides a powerful tool for uncovering hidden patterns in complex fields. Inspired by the Penrose transform and recent developments on coherent D-modules, this project proposes a computational model that simulates black hole–type fields in a discrete space, applying Lagrangian operators and invariant analysis through discrete Radon transforms.

[1]Centro de Estudios Estratégicos del Bajío, Guanajuato, México.
Email: j_carpio@cestbajio.edu.mx
Paper Communicated by Prof. Dr. Francisco Bulnes

To develop a computational model in R that analyses and compares moduli spaces inspired by cohomological relations of black hole–type fields, using discrete Radon transforms and Lagrangian operators to identify invariants stable under broad resolutions, emulating concepts from the Penrose transform.

Then are had the following statements.

1. To implement a discrete grid and a black hole–type field with smoothed singularities and annular structures, representing features of space–time.
2. To apply discrete Lagrangian operators (gradient and Laplacian) to analyse local and global properties of the field.
3. To construct the 2D discrete Radon transform and extract computational invariants (angular peaks) representing the moduli space.
4. To evaluate the equivalence of moduli spaces under Gaussian smoothing, emulating the stability of cohomological classes.
5. To visualise and quantify results in order to verify the consistency and robustness of the extracted invariants.

2 Methods

We consider the following basic definitions on the tools used.

1. **Definition of grid and field:** A 2D discrete space and a black hole–type field with a regularised central core and Gaussian ring are generated.
2. **Application of Lagrangian operators:** Discrete gradient and Laplacian via 2D convolutions to obtain local and global field properties.
3. **Radon transform:** Projection of the field and its derivatives across different angles, generating projection matrices.
4. **Extraction of invariants:** Detection of significant peaks in Radon projections by angle, constituting a computational moduli space.
5. **Broad resolution and comparison:** Gaussian smoothing of the field and repetition of invariant extraction to assess equivalences.
6. **Visualisation and analysis:** Plotting of original images, operators, Radon transforms and detected peaks; calculation of stability statistics of invariants.

3 Result

Complete visualisation of the black hole–type field and its derivatives.

- Radon projections evidencing global field patterns at different angles.
- Extraction of robust invariants preserved even after broad-resolution smoothing.
- Confirmation of equivalences between moduli spaces before and after smoothing.
- Conceptual correlation with Penrose transforms and D-module theory, showing that computational simulations reflect cohomological relations of space–time.

The computational prototype provides an initial framework for numerical analysis and visualisation of moduli spaces and cohomological properties inspired by mathematical physics and field theory. It may be extended to incorporate greater geometric, dynamical, or dimensional complexity, serving as a bridge between theoretical abstraction and numerical representation.

3.1 Explanation of the Image Obtained with the Code

The image generated using R Studio represents the black hole–type field with its regularised core and Gaussian ring, together with the applied gradient and Laplacian operators. The Radon projections display angular peaks that constitute computational invariants of the moduli space.

After applying Gaussian smoothing, the invariants remain stable, demonstrating the equivalence between moduli spaces at different spatial resolutions.

This result evidences how computational simulation reflects persistent properties of space–time geometry, integrating concepts from theoretical physics and applied mathematics.

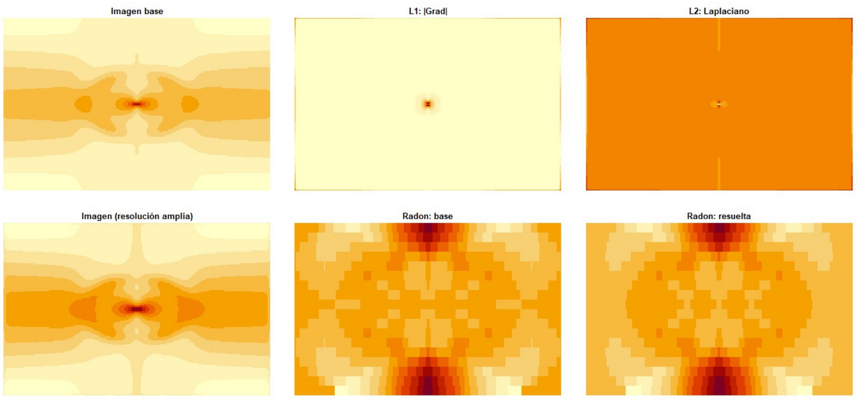

Figure 1: Image obtained using RStudio.

Corollary 7.1"Cohomology and Moduli Corollary of Bulnes" (Authorship of Dr Francisco Bulnes): A Philosophical–Scientific Perspective on Moduli Spaces and String Theory.

The corollary 7.1, defines the moduli space as the set of homological relations between black holes and their distinct cohomological dimensions, showing that these complex structures are manifestations of an underlying order in the universe.

The Radon transform and Lagrangian instanton operators reveal hidden patterns in space–time, and the equivalences among moduli spaces reflect the profound coherence of physical and mathematical reality.

Philosophically, different representations of the universe are equivalent: distinct "languages" describe the same underlying truth. The action of operators in high-dimensional abstract spaces P48, connects the algebraic with the geometric, showing that moduli spaces are coherent and persistent. In heterotic string theory, these equivalences generate the fabric of strings in Calabi–Yau spaces, where cohomological configurations are represented as D-branes.

This perspective consolidates the idea that mathematics reflects the ontological structure of the cosmos, allowing us to understand the harmony and stability of the universe.

3.2 Conclusions: Contribution to the Field of Mathematics Applied to the Universe

This work demonstrates how the abstract concepts of cohomology, the Penrose transform, and coherent D-modules can be translated into robust computational invariants, validating fundamental properties of black hole–type fields.

The integration of numerical simulation and algebraic–geometric theory establishes a framework for exploring the structure of space–time and high-energy physics through replicable computational tools.

Acknowledgements

Special thanks are extended to Dr Francisco Bulnes for his inspiration, guidance, and support in the conceptual development of this work, as well as for his contribution to mathematics and theoretical physics through *Penrose Transform on D-Modules, Moduli Spaces and Field Theory (Advances in Pure Mathematics, 2012).*

References

[1] Bulnes, F. (2012). Penrose Transform on D-Modules, Moduli Spaces and Field Theory. Advances in Pure Mathematics, 2(6), 379–390. http://dx.doi.org/10.4236/apm.2012.26057

Appendix

```
RStudio code used for the generation of the image

## ================================================================

## Proyecto: Proto Radon + Operadores Lagrangianos + Espacio de Modulos

## Inspirado en: relaciones cohomologicas, transformada de Radon,

## operadores lagrangianos y equivalencias tipo Penrose.

## ================================================================

## ----------------------------------

## Utilidades de geometr a y grilla

## ----------------------------------

make_grid <- function(n = 256, xrange = c(-1, 1), yrange = c(-1, 1)) {

x <- seq(xrange[1], xrange[2], length.out = n)

y <- seq(yrange[1], yrange[2], length.out = n)

list(x = x, y = y, n = n)

}

## Campo tipo "black hole"

bh_field <- function(grid, r0 = 0.35, sigma = 0.06, eps = 1e-2) {

n <- grid$n

X <- matrix(rep(grid$x, each = n), n, n)

Y <- matrix(rep(grid$y, times = n), n, n)

R <- sqrt(X^2 + Y^2)
```

```
core <- 1 / sqrt(R^2 + eps^2)

ring <- exp(- (R - r0)^2 / (2 * sigma^2))

img <- scale(core + 2 * ring)

as.matrix(img)
}

## ----------------------------------------

## Operadores lagrangianos discretos

## Gradiente y Laplaciano

## ----------------------------------------

conv2d <- function(img, kernel) {

img <- as.matrix(img)

k <- as.matrix(kernel)

nr <- nrow(img); nc <- ncol(img)

kr <- nrow(k); kc <- ncol(k)

pr <- kr %/% 2; pc <- kc %/% 2

out <- matrix(0, nr, nc)

pad <- matrix(0, nr + 2*pr, nc + 2*pc)

pad[(pr+1):(pr+nr), (pc+1):(pc+nc)] <- img

for (i in 1:nr) {

for (j in 1:nc) {

window <- pad[i:(i+2*pr), j:(j+2*pc)]

out[i, j] <- sum(window * k)

}

}

out

}

kern_laplace <- matrix(c(0,1,0,1,-4,1,0,1,0),3,3,byrow=TRUE)

kern_dx      <- matrix(c(0,0,0,-1,0,1,0,0,0),3,3,byrow=TRUE)
```

```
kern_dy         <- t(kern_dx)

L1_gradient_mag <- function(img) {
gx <- conv2d(img, kern_dx)
gy <- conv2d(img, kern_dy)
sqrt(gx^2 + gy^2)
}

L2_laplacian <- function(img) {
conv2d(img, kern_laplace)
}

## ------------------------------------------------
## Transformada de Radon discreta
## ------------------------------------------------
radon_discrete <- function(img, thetas = seq(0,179,by=1), nrho=180) {
img <- as.matrix(img)
n <- nrow(img)
cx <- (n+1)/2; cy <- (n+1)/2
Rmax <- sqrt(2)*(n/2)
rhos <- seq(-Rmax, Rmax, length.out = nrho)
sincos <- cbind(sin(thetas*pi/180), cos(thetas*pi/180))
proj <- matrix(0, nrow=length(rhos), ncol=length(thetas),
dimnames = list(NULL, paste0("th=",thetas)))

for (tix in seq_along(thetas)) {
s <- sincos[tix,1]; c <- sincos[tix,2]
for (rix in seq_along(rhos)) {
rho <- rhos[rix]
```

```
t_vals <- seq(-Rmax, Rmax, length.out=n)
x <- rho*c - t_vals*s + cx
y <- rho*s + t_vals*c + cy
xi <- round(x); yi <- round(y)
mask <- xi>=1 & xi<=n & yi>=1 & yi<=n
if(any(mask)){
idx <- xi[mask] + (yi[mask]-1)*n
proj[rix,tix] <- sum(img[idx])
}
}
}
attr(proj,"rhos") <- rhos
attr(proj,"thetas") <- thetas
proj
}

## ————————————————————————————————
## Extracci n de invariantes del espacio de m dulos
## ————————————————————————————————
extract_moduli_invariants <- function(R, peak_prom=0.25){
thetas <- attr(R,"thetas")
rhos   <- attr(R,"rhos")
inv <- lapply(seq_along(thetas), function(j){
v <- R[,j]
v <- as.numeric(v)
v <- v / max(1e-9, max(v))
thr <- peak_prom
idx <- which(v > thr)
```

```r
peaks <- idx[which(diff(c(-Inf, v[idx]))>0 & diff(c(v[idx],-Inf))<0)]
list(theta=thetas[j], rho_peaks=rhos[idx[peaks]])
})
inv
}

moduli_equivalent <- function(invA, invB, tol=2){
thA <- sapply(invA,function(x)x$theta)
thB <- sapply(invB,function(x)x$theta)
if(!all(thA==thB)) return(FALSE)
ok <- logical(length(invA))
for(k in seq_along(invA)){
nA <- length(invA[[k]]$rho_peaks)
nB <- length(invB[[k]]$rho_peaks)
ok[k] <- abs(nA-nB)<=tol
}
mean(ok) > 0.97
}

## ----------------------------------------
## Demo computacional
## ----------------------------------------
set.seed(7)
grid <- make_grid(n=192)
img0 <- bh_field(grid)

grad_mag <- L1_gradient_mag(img0)
lap      <- L2_laplacian(img0)

R_base <- radon_discrete(img0, thetas=seq(0,170,by=10), nrho=220)
```

```
R_grad <- radon_discrete(grad_mag, thetas=seq(0,170,by=10), nrho=220)
R_lap  <- radon_discrete(lap, thetas=seq(0,170,by=10), nrho=220)

## Resoluci n amplia (suavizado)
gauss_kernel <- function(sigma=1){
r <- ceiling(3*sigma)
x <- -r:r
g <- exp(-x^2/(2*sigma^2))
g <- g/sum(g)
outer(g,g)
}

smooth2d <- function(img, sigma=1.2, passes=2){
K <- gauss_kernel(sigma)
out <- img
for(i in 1:passes) out <- conv2d(out,K)
out
}

img_res <- smooth2d(img0,1.1,2)
R_res   <- radon_discrete(img_res, thetas=seq(0,170,by=10), nrho=220)

inv_base <- extract_moduli_invariants(R_base, peak_prom=0.35)
inv_res  <- extract_moduli_invariants(R_res, peak_prom=0.35)

equiv <- moduli_equivalent(inv_base, inv_res, tol=2)
cat(sprintf("\n Equivalencia de moduli (base vs. resuelto)? %s\n",
ifelse(equiv,"SI","NO")))

## Visualizaci n
```

```r
par(mfrow=c(2,3), mar=c(2,2,2,1))
image(t(apply(img0,2,rev)), main="Imagen base", axes=FALSE)
image(t(apply(grad_mag,2,rev)), main="L1: |Grad|", axes=FALSE)
image(t(apply(lap,2,rev)), main="L2: Laplaciano", axes=FALSE)
image(t(apply(img_res,2,rev)), main="Imagen (resoluci n amplia)", axes=FALSE)
image(R_base, main="Radon: base", axes=FALSE)
image(R_res,  main="Radon: resuelta", axes=FALSE)
par(mfrow=c(1,1))

## Reporte breve
npeaks_per_angle <- function(inv) sapply(inv, function(z) length(z$rho_peaks))
cat("Promedio de picos (Radon) por  ngulo   base:",
mean(npeaks_per_angle(inv_base)), "\n")
cat("Promedio de picos (Radon) por  ngulo   resuelta:",
mean(npeaks_per_angle(inv_res)), "\n")
```